Existence of equilibrium

- Is there a p^* that clears all markets?

- Equilibrium exists if the technology of the firm is convex and it does not matter if the aggregate technology of the economy is convex.

- If each consumer's demand function $x_i(p, p\omega_i)$ is homogeneous of degree zero $\Rightarrow$ The aggregate excess demand function $z(p)$ is also homogeneous of degree zero, because the sum of homogeneous functions is also homogeneous.

$$\Downarrow$$

- We can normalize (fix) prices at an arbitrary level and express demand in terms of relative prices
- G.Debreu, 1974: any continuous function that satisfies Walra's law is an excess demand function for some economy, i.e. there is no restrictions on aggregate demand behavior

Uniqueness

- Is Walrasian equilibrium unique?

- If uniqueness is not achievable, the next-best property is local uniqueness.

- An equilibrium price vector is locally unique (isolated) if we cannot find another normalized price vector arbitrarily close to it.

- If an economy has only a finite number of normalized price equilibria $\Rightarrow$ The equilibrium must be locally unique.

Uniqueness

- Three conditions that ensure uniqueness (global):
  - 1. If all goods are gross substitutes at all prices $\Rightarrow$ if p* is an equilibrium price vector, it is the unique equilibrium price vector. Two goods are **gross substitutes** if an increase in price of one good causes an increase in the excess demand for another good (Walrasian demand) gr. 16
  - 2. If there is no trade an equilibrium (initial endowment allocation constitutes a Walrasian equilibrium allocation) $\Rightarrow$ this is the unique equilibrium allocation
  - 3. Index analysis of the Jacobian matrix: if the negative of the Jacobian matrix of the excess supply function has positive determinant at all equilibria, then there is only one equilibrium. $\Rightarrow$ if we can attach sign to the determinant of the Jacobian matrix of the equilibrium equations at any solution point, then the equilibrium must be unique. (sign uniformity across equilibria is impossible if there is multiplicity)

Conditions

- If we have a system of *M* equations and *N* unknowns, i.e. $(N\text{-}M)$ degrees of freedom:

  \- $M < N \Rightarrow$ we can express the values of *M* variables that solve the *M* equations as a function of the $(N\text{-}M)$ remaining variables, i.e. the solution will be a set.

  \- $M = N \Rightarrow$ equilibria must be locally unique

  \- $M > N \Rightarrow$ no solution if equations are nonequivalent (the system is overdetermined)

- M and N determine number of columns and rows in a Jacobian matrix respectively.
- The first proof of existence of general equilibrium was formulated by McKenzie, Arrow & Debreu (1954). The issue of how to actually find equilibria was first considered by H. Scarf in 1973. By now, a variety of useful techniques are available. (gr. 17)

Tâtonnement

- We’ve talked about Walras equilibrium conditions, existence and uniqueness. We did not talk about how to find an equilibrium.
- Where does the equilibrium price „come from”, since everyone is a price-taker.
- Would a „marketeer”, who does not know the form excess demand functions, be able to arrive at an equilibrium?
- Walras himself was interested in these question. He used the term „ tâtonnement” (groping) for the process of arriving at an equilibrium.

Tâtonnement Process

- A natural tatonnement algorithm for finding an equilibrium is to start from some price vector, and then increase (decrease) the prices of the goods, for which the excess demand (supply) is positive
- Technically, we follow a process described by a differential equation

$$dp_k / dt = c_k z_k(p)$$

  where dp_k/dt is the rate of change of the price for the kth good and $c_k>0$ is a constant affecting the speed of adjustment
- It is a tentative trial & error process taking place in fictional time (t is not real time) and run by an abstract market agent bent on finding the equilibrium level of prices.
- **Calibration** means positioning of model parameters and exogenous variables, such that the model replicates the data of the base period. The economy under consideration is assumed to be in equilibrium, a so-called **‘benchmark’ equilibrium** (base period).

Tâtonnement Stability

- An equilibrium p^* is **locally stable**, if the dynamic process causes the prices to converge (coming closer) to the vector proportional to p^*
- The equilibrium is **locally totally unstable**, if any disturbance causes the relative prices to diverge from p^*
- Uniqueness by itself does not imply stability.
- There is **system stability** if for any initial prices, the dynamic process converges to a price vector that is proportional to some equilibrium vector
- **Global stability** means the convergence of any price trajectory to Walrasian equilibrium

Tâtonnement Stability

- If an economy has more than 2 goods, we may have a situation, where an equilibrium is neither locally stable nor locally totally unstable (we may have saddle points)
- Economies that have a unique equilibrium are (globally) stable systems.
- To get global stability results we have to assume special conditions on demand functions. The value of the results will then depend on the economic naturalness of the conditions assumed.

Size of economy

- A central justification of the price-taking hypothesis is the assumption that every economic agent constitutes an insignificant part of the whole economy (no matter of economy size).

$$\Downarrow$$

- It cannot be satisfied in our models, because we allow for one or two consumers
- An increase in the size of the economy insure existence of a **near-equilibrium** (an allocation and price vector that is close to satisfying the conditions of an equilibrium) for any $p>>0$.
- The economy will possess a near-equilibrium if (i) the size of the consumption side of the economy is large relative to the maximal size of a single firm and (ii) a small proportion of consumers may display a nonconvexity at p^*.